Flow of Water in Curved Pipes.

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(Communicated by Sir J. Larmor, Sec. R.S. Received April 25,—Read June 2, 1910.)

Preliminary Experiments.—The author has for several years been engaged in experimentally investigating the resistance to the flow of fluids in metal pipes, both straight and curved. In order to study the effect of change of curvature, several experiments have been made on the flow of water in flexible pressure tubing with thick walls—a composite of rubber and canvas—the inner portion being red rubber surrounded by canvas, the canvas being covered on the outside with grey rubber.

Dimensions of tube:—Outside diameter 0·81 cm.; inside diameter 0·3684 cm. (calculated from the volume of the tube.)

A flexible tube was selected for the experiments because of the variation in the resistance to the flow of water in metal tubes which appear to be in the same surface condition; it was thought that if the same tube were used for a series of experiments at different curvatures this variation of resistance, due to the variation in roughness of the surfaces, would not occur. But another cause of difficulty was introduced, viz.: the slow recovery from strain of the rubber tubing, both after change of form due to coiling and after changes in pressure during the flow of water.

Two preliminary series of experiments were made, before the fuller investigation was undertaken. In the first series a pear-shaped glass vessel of 940 c.c. capacity was used as a containing vessel for the supply of water, the top being open to the atmosphere, and a short piece of tubing, about five times the area of the flexible tube, connected the latter to the containing vessel.

For straight-tube flow, the flexible tube was laid on an inclined plane; for coiled-tube flow, the tube was coiled around cylinders; but in all the experiments the angle of the helix was kept constant.

In the second series, practically the same arrangement was adopted except that a special containing vessel was employed which was designed so that the head of water was maintained constant throughout the series, and there was a glass connecting tube between the containing vessel and the experimental tube.

In the first series the time required for the flow of a known volume was recorded. In the second series, the quantity of water discharged in

200 seconds was recorded. In both series the water ran freely from the end of the tube into the measuring vessel. The method of experimenting was similar to that described on p. 110.

These experiments showed that the change from straight-tube flow to coiled-tube flow was accompanied by an increasing resistance, which when expressed in terms of time of flow and curvature, after correcting for diminution in the area of the tube due to coiling, could, for values of r/R between 0 and 0·06, be represented by an equation of the form

$$\{(t-t_0)/t_0\}^n = \theta\mathrm{R}^{-1} = \text{constant} \times \text{curvature}.$$

Where t is time of flow in coiled tube,
t_0 is time of flow in normal straight tube = a constant,
r is radius of tube,
R is radius of curvature of the coil,
θ is constant for constant pressure-head,
$n = 1{\cdot}8$ nearly.

Extension of Experiments.—In the preliminary experiments the pressure-head on the water was nearly constant, and consequently the velocity of flow of the water was limited in range. In the fuller experiments now to be described the pressure-head could be varied and accurately adjusted so as to give velocities of flow both below and above the critical velocity; the change in the law of resistance to flow could thus be studied, when the flow changed from steady to unsteady motion in the straight tube and could be compared with the results obtained in the coiled tube.

The primary object of the investigation was to determine the variation in the resistance to the flow of water due to the curvature of the tube length, but as the area of the tube section diminished with an increase of curvature it was necessary to experiment separately on the effect produced on the flow of water in a straight tube by a variation in sectional form and area; hence two investigations were carried out, each consisting of several series of experiments.

A. *Coiled Tube.*—The determination of the combined effect of the variation in curvature of the length of the tube and of the form and area of section.

B. *Oval Tube.*—The determination of the effect of the variation in the form of section of the straight tube.

Description of the Apparatus.—A cylindrical hydraulic tank has been erected in the hydraulics laboratory of the Hartley University College; the tank, which is 2 feet (61 cm.) diameter, gives a direct head of 36 feet (1,100 cm.) and is constructed so that arrangements can be made to increase the pressure by means of compressed air; the general arrangement is shown in fig. 1.

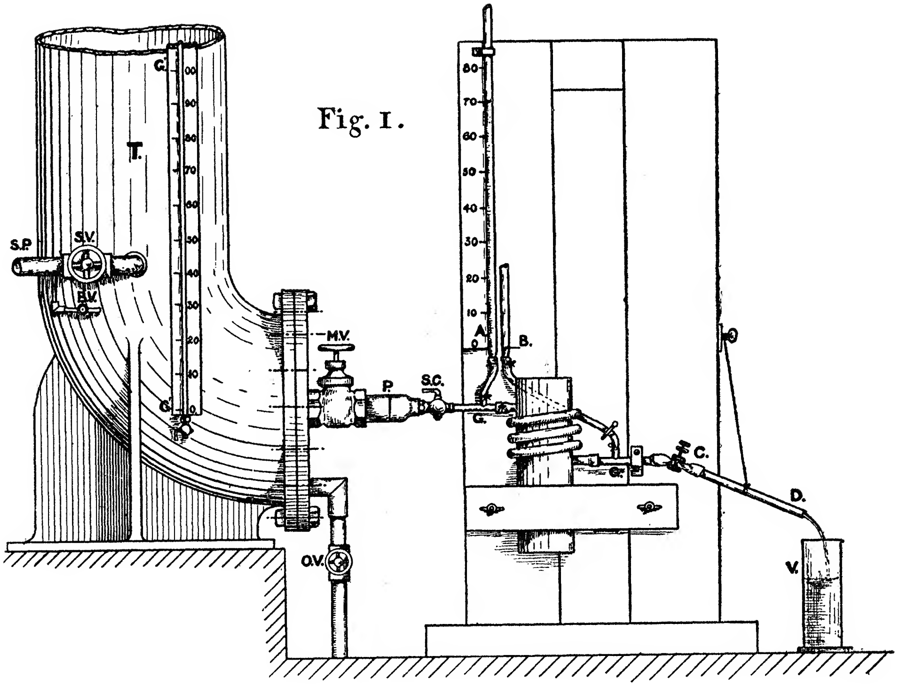
Fig. 1.

The supply of water to the tank T is taken from the town mains through a two-inch supply pipe S.P.; the flow of water into the tank is regulated by the supply valve S.V., while very fine adjustments of the level of the water in the tank can be made by the aid of a small by-pass valve B.V. which is fitted to the supply pipe. An outlet valve O.V. is provided for emptying the tank.

The head of water in the tank for its entire height can be read on gauge glasses G′, G′, attached to the tank. A short pipe P connects the tank with the experimental tube; between the stop-cock S.C. and the tank is the "full-way" main sluice valve M.V. In each experiment a brass gauge pipe G was screwed into the stop-cock S.C.; this pipe brought the flexible tube into communication with the tank.

A. *Coiled Tube Experiments.*—The arrangement adopted in the coiled tube experiments was as follows:—

Two brass pipes G, G, of the same bore as the flexible tube were attached to the ends of the tube, gauge holes (1 mm. diameter) drilled at right angles to the pipes communicated with the gauge glasses A and B, fig. 1.

The flexible tube was coiled around wooden cylinders which had been turned to the diameters necessary to give a whole number of coils for a given length of tube; the positions of the gauge portions G, G, could thus be the same in all the experiments. In each experiment the supporting cylinder was fixed on a stand, the gauge portions of the tube were horizontal, and the gauges placed together as shown in the sketch.

The water in the gauge B (at the outlet end) was maintained at a constant level; the flow through the tube was regulated by means of a pinch-cock C. For fine adjustments of level the end of the drain tube D could be raised or lowered by a simple device; the loss of head between the gauges was given by the reading of the gauge A above the datum line at B.

For the lower pressures, up to a head of 200 cm., the loss of head was read directly in centimetres of water, above this head a specially constructed mercurial gauge was used.

Method of Experimenting.—The apparatus was set up and after allowing the water to flow through the tube for several minutes till the *régime* was found to become steady the inlet valve to the tank and the outlet pinch-cock C were adjusted until the required difference in the level was indicated by the gauges; when this difference became permanent the outlet water was caught in a vessel V, and the quantity discharged in a definite time (usually 100 seconds) was measured. The experiment was then repeated in the same way at other pressures until the series was completed.

A series was usually commenced at a low pressure, with a difference of head of a few millimetres, and continued under an increasing head until the difference of head was about 1000 cm.; in some of the experiments, however, the series was commenced with the pressure due to a full tank.

The experimental results obtained were tabulated as in columns (2), (5), and (6) of the accompanying Table I (No. 0c), which gives the results for the series when the tube rested on an inclined plane; the plotted results gave curves practically identical with the curves obtained on the same day with a straight tube in which both ends were kept at the same level. These experiments showed that the loss of head due to the changes from the horizontal to the inclined direction could be neglected; the standard of comparison was, however, that of the flow through the inclined tube 0c.

Table II (No. 10c) is a similar table for 10 coils. In column (7) a correction is made for temperature. This correction is based on Poiseuille's formula* for increase of flow with increase of temperature in a straight capillary tube; in the table it is assumed that Poiseuille's constants hold good for coiled-tube flow.

The coiled portion of the flexible tube was in all cases five-sixths of the length between the gauges, that is to say, one-sixth of the tube length was always joined on tangentially in the normal condition; the experimental values were corrected in the following manner.

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* $Q \propto (1 + 0{\cdot}03368\,T + 0{\cdot}000221\,T^2)$, where T is temperature Centigrade.

Table I.—Inclined Tube. No. 0c.

Area of tube section, 0·1066 sq. cm.
Height of datum line from lower gauge pipe, 28·8 cm.
Distance between horizontal gauge pipes, 10·0 cm.
Length of tube between gauges, 97·8 cm.
Length of inclined part of tube, 81·5 cm.

(1) No.	(2) Head in cm. of water.	(3) Loss of head in horizontal part.	(4) (2)—(3) Loss in inclined part.	(5) c.c. per sec.	(6) Temp. Centigrade. °	(7) c.c. per sec. for 14° C.
1	1·00	0·166	0·833	0·383	15·2	0·37
2	2·00	0·333	1·666	0·750	15·0	0·73
3	4·20	0·700	3·500	1·30	14·0	1·30
4	9·40	1·566	7·833	2·60	14·0	2·60
5	11·30	1·883	9·416	2·97	14·0	2·97
6	15·20	2·533	12·666	3·76	13·8	3·78
7	18·60	3·100	15·500	4·43	13·8	4·45
8	21·80	3·633	18·166	5·00	13·8	5·02
9	25·00	4·166	20·833	5·62	13·8	5·64
10	30·00	5·000	25·000	6·40	13·8	6·43
11	35·10	5·850	29·250	7·08	13·7	7·13
12	40·00	6·666	33·333	7·55	13·6	7·60
13	50·00	8·333	41·666	8·11	13·5	8·20
14	60·00	10·000	50·000	8·78	13·5	8·88
15	70·00	11·666	58·333	9·40	13·5	9·51
16	80·40	13·400	67·000	10·15	13·5	10·26
17	100·00	16·666	83·333	11·40	13·4	11·55
18	150·00	25·000	125·000	14·22	13·3	14·46
19	214·00	35·660	178·330	17·27	13·3	17·57
20	300·00	50·000	250·000	20·90	13·0	21·45
21	400·00	66·660	333·330	24·45	13·0	25·10
22	500·00	83·330	416·660	27·84	12·9	28·70
23	700·00	116·660	583·330	33·40	12·8	34·40
24	944·00	157·330	786·660	38·90	12·6	40·40

A compensating curve giving the loss of head for one-sixth of the pipe in its normal condition was plotted to a base line of discharge in cubic centimetres per second, the experimental readings in cubic centimetres per second were compared with the compensating curve, and the loss of head reduced by the amount given by the curve. The corrected values were taken as the loss in the coiled portion of the tube; this is shown in columns (3) and (4) of the tables.

Calibration of the Coiled Tube.—By means of a graduated burette attached to the tube the quantity of water required to fill the straight tube was determined, and also the quantity displaced at the various curvatures of the experiments; the area of the section corresponding to any particular curvature was calculated from the experimental data obtained.

The results are given in columns (1) to (4) of Table III, and the values of A/a are plotted on the curve aAA' in fig. 3.

Table II.—Ten Coils. No. 10c.

Area of tube section, 0·0735 sq. cm.
Diameter of coil: internal, 1·9 cm.; external, 3·2 cm.
Height of datum line from lower gauge pipe, 28·8 cm.
Distance between horizontal gauge pipes, 10·0 cm.
Length of tube between gauges, 97·8 cm.
Length of coiled part of tube, 81·5 cm.

(1) No.	(2) Head in cm. of water.	(3) Loss of head in straight part.	(4) (2)—(3) Loss in coiled part.	(5) c.c. of water per sec.	(6) Temp. Centigrade.	(7) c.c. per sec. for 14° C.
					°	
1	1082·0	48·0	1034·0	20·94	13·2	21·40
2	966·0	42·5	923·5	19·72	13·2	20·16
3	857·0	37·5	819·5	18·53	13·2	18·92
4	730·0	32·0	698·0	16·94	13·2	17·31
5	585·0	24·5	560·5	14·80	13·3	15·10
6	448·0	18·1	429·9	12·60	13·6	12·73
7	330·0	12·3	317·7	10·34	13·8	10·40
8	237·0	8·0	229·0	8·32	14·0	8·32
9	186·5	5·3	181·2	7·15	14·1	7·10
10	120·0	3·3	116·7	5·24	14·2	5·21
11	93·5	2·6	90·9	4·40	14·2	4·38
12	72·4	2·1	70·3	3·70	14·5	3·65
13	47·8	1·45	46·35	2·75	14·7	2·70
14	29·65	0·97	28·68	1·93	14·9	1·88
15	16·6	0·58	16·02	1·23	15·0	1·20
16	7·9	0·28	7·62	0·68	15·5	0·654
17	5·89	0·21	5·68	0·532	15·6	0·510
18	2·38	0·08	2·3	0·248	16·0	0·233
19	1·45	0·05	1·4	0·160	16·1	0·150
20	0·53	0·02	0·51	0·0683	16·2	0·064

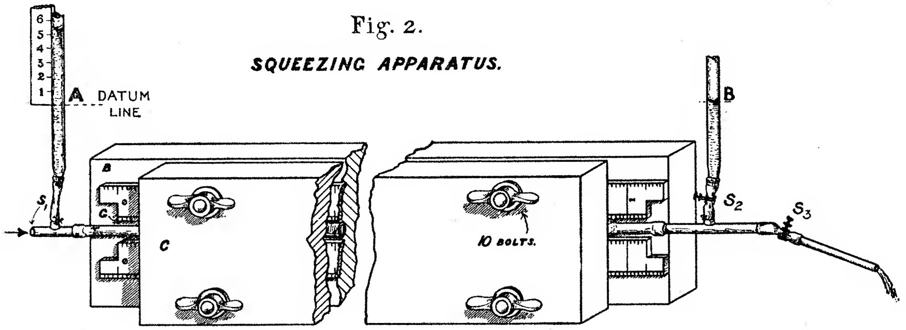
Table III.

(1) No. of coils.	(2) Curvature R^{-1}.	(3) Area A.	(4) A/a.	h/l average from		
				(5) 0·03 to 0·3.	(6) 0·3 to 3·0.	(7) 3·0 to 10·0.
				n.	*n.*	*n.*
0	0·0	0·1066	1·0	1·20	—	1·81
½	0·0345	0·1066	1·0	1·31	1·55	—
1	0·0782	0·1065	0·999	1·30	1·50	1·90
2	0·1570	0·1057	0·990	1·34	1·51	1·89
3	0·2330	0·1047	0·982	1·35	1·49	1·78
4	0·3090	0·1025	0·962	1·36	1·48	1·80
5	0·3900	0·0996	0·934	1·36	1·48	1·74
6	0·4680	0·0960	0·900	1·35	1·46	1·78
7	0·5440	0·0907	0·850	1·32	1·45	1·74
8	0·6250	0·0850	0·796	1·31	1·44	1·66
9	0·7110	0·0790	0·741	1·29	1·42	1·63
10	0·7850	0·0735	0·689	1·23	1·41	1·67

B. *Oval Straight Tube.*—In order to study the effect of diminishing the area of section in the case of the straight tube a special form of squeezer was made (fig. 2), which consisted of two flat pieces of wood. In one of these, the block B, a groove G was formed for the reception of the tube T; the cover C was bolted to B by 10 bolts, the effective depth of the groove being adjusted by the insertion of thin strips of rolled metal.

Fig. 2.

SQUEEZING APPARATUS.



(*a*) Determination of the area of section.

The apparatus was set up in position, the tube being quite free in the groove, water was allowed to run until a constant *régime* was obtained, the ends of the tube and the outer gauge were then stopped by means of the stop-cocks at S_1, S_2, and S_3, and a graduated tube was substituted for the upper gauge. The cover C was then screwed tightly against the block B, and the diminution in volume read in the graduated tube, the decrease in mean area being calculated from the data obtained. The results are given in Table IV.

(*b*) Loss of "head" at different velocities.

The same method of working was adopted as in the corresponding experiments on the coiled tube, *q.v.*

Table IV.

(1) Experiment mark.	(2) Area A. Experimental.	(3) A/*a*.	Values of *n*		Remarks.
			(4) Below C.V.	(5) Above C.V.	
A	0·1066	1·000	1·20	1·87	The reduction in area for D, E, F is beyond the limit of the experiments on the coiled tube. Compare the values of A/*a* in Column (3) with the values in Column (4) of Table III.
B	0·0882	0·826	1·08	1·92	
C	0·0735	0·688	1·08	1·93	
D	0·0637	0·597	1·08	1·94	
E	0·0550	0·515	1·04	1·86	
F	0·0516	0·483	1·12	1·58	

As in the coiled-tube experiments the part of the tube which was subjected to change of section was five-sixths of the total length of the tube, and corrections were made in the same way as is described on p. 111.

Results of the Experiments.—The results of all the experiments have been tabulated as in Tables I and II, and have been carefully plotted on squared paper. The curves which are reproduced in figs. 4 and 5 illustrate the method adopted.

In fig. 4 the abscissæ represent the quantities of flow in cubic centimetres per second (column (7) in Table I), the ordinates represent the losses of head in centimetres (column (4) in Table I). The curve 0*a*A is for the straight tube (No. 0c, Table I). The curve 01 is for the coiled tube (No. 1, one coil).

In fig. 5 the logarithms of the flow in cubic centimetres per second are plotted as abscissæ, and the logarithms of the loss of head as ordinates. The full line curve 0*a*A is for the straight tube 0c. The other full line curves are for $\frac{1}{2}$, 1, 3, etc., coils. The dotted line curves BB, CC, DD, etc., are for the straight tube when it has been made oval in section by squeezing.

It will be seen from fig. 5 that for a given loss of head there is a very great increase in resistance, or diminution in the quantity of flow, as the number of coils increases, or as the curvature increases; the diminished flow, as is shown by the logarithmic curve, is most marked at points which are below the critical velocity in the straight tube. It may be noted, in passing, that there is no "critical velocity" region in the coiled tubes; but this and other peculiarities of these curves are dealt with below.

It has been shown that there is a reduction in area of the tube as the radius of the curvature decreases. But there is no appreciable reduction in area when the tube is slightly coiled as in "half coil" and "one coil," yet in these cases the diminution in flow is very great at discharges corresponding to the critical velocity in the straight tube; this is shown by the lines O*a*A and 1, 1 in fig. 5 and also in the corresponding diagram (fig. 4) for discharges between 5 and 10 c.c. per second.

Absence of "Critical Velocity" in the Coiled Tube.—In all the coiled-tube experiments the change in slope of the curves, as shown by the logarithmic plotting, is gradual. This change in slope is also seen in the values of n in columns (5), (6) and (7) of Table III, where n is the index of v in the formula $s = h/l = \mathrm{K}v^n/m$, s or h/l is the hydraulic gradient, v the velocity of flow, m the hydraulic mean radius, and K is a constant.

The difference in the character of the curves obtained for straight-tube flow and for coiled-tube flow is apparent in the two curves shown in fig. 4. This difference is even more marked in fig. 5, where O*a*A is plotted for a straight tube and I.I for the tube when formed into a coil of large radius, both tubes

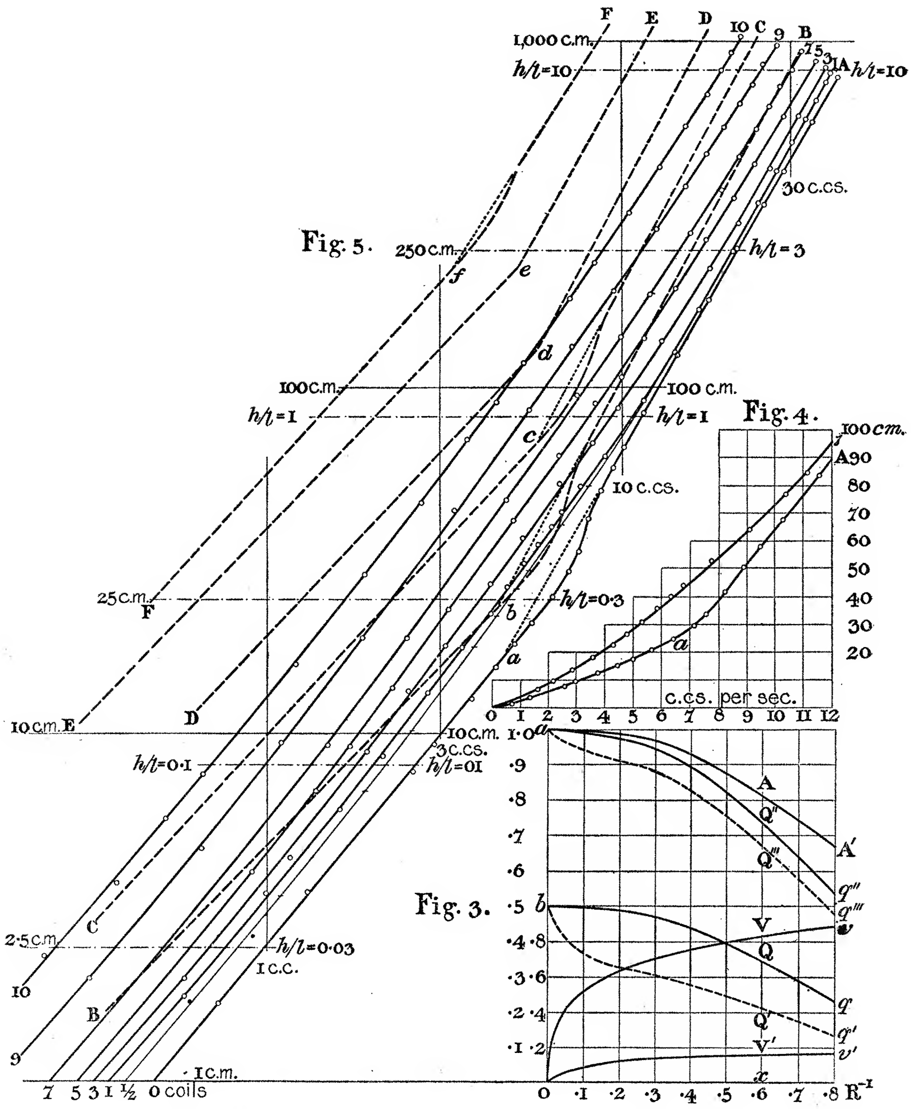


Fig. 5.

Fig. 4.

Fig. 3.

have practically the same form and area of section. The curve C*c*C for a straight oval tube may, in the same way, be compared with the curve for ten coils, both these curves being for tubes of the same form and area of section.

The dotted-line curves in fig. 5 show that when the straight tube is made oval by squeezing the curves are all of the same general form as the curve O*a*A. The full-line curves show that even a small curvature of the tube length tends to modify the "critical velocity," and to make the increase in slope gradual.

Ratio of the Flow in a Coiled Tube to the Flow in a Straight Tube of the same Form and Area of Section.—In fig. 3 the abscissæ represent curvature R^{-1}. For a given hydraulic gradient the ordinate $0b$ represents unit quantity of water flowing through a normal straight tube; the ordinates of the dotted-line curve $bQ'q'$ represent the quantity of flow Q' through the tube when it is coiled; the ordinates of the full-line curve bQq represent the quantity of flow Q through the tube when it is straight; the time of flow is the same in each case. Corresponding to any given curvature R^{-1}, the cross-section of the tube both when coiled and when straight is of the same form, as well as of the same area A.

The lower curves $bQ'q'$, bQq, are for velocities of flow less than the critical velocity in the straight tube.

The upper curves $aQ'''q'''$, $aQ''q''$, are for velocities of flow greater than the critical velocity, and in the diagram the ordinates of the upper curves are drawn to twice the scale of the lower curves.

$xQ'/xQ = Q'/Q =$ flow in coiled tube ÷ flow in straight tube. Then $(Q-Q')/Q = \Delta Q/Q =$ loss of flow due to coiling ÷ flow in a straight tube of the same form and area as the coiled tube. And since Q varies directly as V (the velocity), the area being constant, $\Delta Q/Q = \Delta V/V =$ loss of velocity due to coiling ÷ velocity in a straight tube.

The ordinates of the curve OVv represent the ratio $\Delta Q/Q$ or $\Delta V/V$, for values of velocity below the critical velocity. $OV'v'$ is the corresponding curve for values of $\Delta V/V$ above the critical velocity.

These curves are represented by the equation $(\Delta V/V)^n = CR^{-1}$, where C is a constant for a constant hydraulic gradient.

The values of $\Delta V/V$ have been plotted for several hydraulic gradients; all the curves have the same general character, except that at velocities just above the higher critical velocity there is a change in the form of the curves near the origin, *i.e.* for small curvatures of the tube length.

The values of $\Delta V/V$ increase with increase of velocity up to the critical velocity, where a maximum is reached. At high velocities the form of the curves is the same as at low velocities, but the ratio $\Delta V/V$ diminishes with increase of velocity.

The value of n is 3 at the critical velocity, it becomes less than 2 just above the critical velocity, then increases to 3·3 at the higher velocities of the experiments.

Summary of Results.—The results of the experiments may be summarised as follows:—

(*a*) The flow in straight flexible tubes obeys the laws of flow in metal tubes as investigated by Prof. Osborne Reynolds ('Phil. Trans.,' 1883). In

Reynolds' experiments at velocities below the "critical velocity" the flow is proportional to the velocity. In the author's experiments the flow is proportional to the velocity raised to the power n; when the tube was free, n varied from 1·18 to 1·2; when the tube was squeezed in a straight groove, n became 1·08 to 1·04. This discrepancy is accounted for in the foregoing investigation, for it is shown that a curvature of the tube length increases the value of n; the flexible tube when not confined in a groove was sinuous, this sinuosity of the tube length increases n.

In other experiments on metal tubes, at velocities below the critical velocity, the author has found that whilst $n = 1$ for straight tubes, the effect of introducing angle bends in the tube length has been to increase n to 1·1.

The velocity at which the flow becomes turbulent occurs in the normal straight tube at the value given by Reynolds' formula

$$\mathrm{V}_c = \mathrm{P}/278\,\mathrm{D}, \quad \mathrm{P} = (1 + 0{\cdot}0336\,\mathrm{T} + 0{\cdot}000221\,\mathrm{T}^2)^{-1},$$

where D is diameter of the tube in metres and T is temperature of the water.

The dotted curves in fig. 5 illustrate a peculiarity observed by Reynolds; it will be seen from the figure that the critical velocity region is different in the curves A, B, C, and F from the critical velocity region in D and E; the former were obtained from experiments at gradually increasing velocity, the latter at gradually decreasing velocity.

(*b*) The "critical velocity" region is present in all the squeezed tube experiments, even when the tube is flattened sufficiently to reduce the area to less than one-half the original area (as is shown by the set of curves A, B, C, D, E, F, in fig. 5). And the flow obeys the index law both below and above the critical velocity; that is to say, in the logarithmic plotting the lines below and above the critical velocity are both straight.

In all the coiled tube experiments, on the other hand, the value of the index n gradually increases with the velocity of flow, and there is an entire absence of the "critical velocity" region. The lines in the logarithmic plotting have a *gradually* increasing slope from the low velocities to the higher velocities of flow, as is shown by the set of curves in fig. 5.

The experiments show conclusively that this absence of the critical velocity region is not due to the variation in sectional form or area.

(*c*) The increased resistance expressed in terms of the loss in quantity discharged, or in loss of velocity for a given loss of pressure, is given approximately by the formula

$$(\Delta\mathrm{Q}/\mathrm{Q})^n = (\Delta\mathrm{V}/\mathrm{V})^n = \mathrm{CR}^{-1}.$$

(*d*) The slow recovery of the flexible tube from the strain due to deformation caused by squeezing, or by coiling, adversely affected the experiments; but the results of the time lag were not sufficiently serious to affect materially the general deductions which have been made.

The author is continuing the investigation, in the hydraulics laboratory of the Hartley University College, with specially made metal tube of various forms of section, with the object of determining the commencement of turbulent flow in coiled tubes. He is also investigating the effect of the ratio of area to perimeter in oval tubes on the flow and critical velocity.